Midterm #1 Study Guide
Math 156 (Calculus I), Fall 2024

1. Basics (domain/range, what graph looks like, etc.) for standard functions [§1.1, 1.2, 1.4, 1.5]
   (a) algebraic functions: power functions (like x^3), root functions (like $\sqrt{x}$), polynomials (like $x^2 - 3x + 1$), rational functions (like $\frac{x^2-1}{x+5}$)
   (b) trigonometric functions (like $\sin(x)$ and $\cos(x)$)
   (c) exponential functions (like e^x) and logarithmic functions (like $\ln(x)$)
   (d) piecewise functions (like absolute value $|x|$)
2. Algebraic operations on functions as geometric operations on graphs [§1.3]
   (a) translation (up/down & left/right), stretching (horiz. & vert.), reflection (over axes)
   (b) symmetry under these operations, especially even and odd functions
3. How to make new functions from old functions $f(x), g(x)$ [§1.3]
   (a) sum $(f + g)$, difference $(f - g)$, scaling (cf), product (fg), quotient (f/g)
   (b) composition of functions: $(f \circ g)(x) = f(g(x))$
4. Inverse functions $f = g^{-1}$ [§1.5]
   (a) especially exponential and logarithmic functions
   (b) graph of inverse function is reflection across line $y = x$
5. Intuitive definition of limit and basic reasons why a limit might not exist [§2.2]
   (a) $\lim_{x\to a} f(x) = L$ means can make $f(x)$ arbitrarily close to L by making $x \neq a$ close to a
   (b) one-sided limits $\lim_{x\to a^\pm} f(x)$: they must agree for usual (two-sided) limit to exist
   (c) $\lim_{x\to a} f(x) = \pm\infty$ counts as the limit not existing
6. How to compute limits using the limit laws [§2.3, 2.5]
   (a) sum $(f + g)$, difference $(f - g)$, scaling (cf), product (fg), quotient (f/g) limit laws
   (b) how to deal with "$\frac{0}{0}$" by cancelling factors
   (c) continuous functions (pushing limit thru, and direct substitution a.k.a. "plugging in")
7. Limits at infinity and limits equal to infinity [§2.2, 2.6]
   (a) limits at $\pm\infty$ = horizontal asymptotes (typical example: $\lim_{x\to-\infty} e^x = 0$)
   (b) $\pm\infty$-valued limits = vertical asymptotes (typical example: $\lim_{x\to 0^+} 1/x = \infty$)
8. The definition(s) of derivative [§2.1, 2.7, 2.8]
   (a) derivative as slope of the tangent to the curve $y = f(x)$ at a point $x = a$
   (b) derivative as a limit $f'(a) = \lim\limits_{x\to a} \dfrac{f(x) - f(a)}{x - a}$