Midterm #2, 11/4
Math 156 (Calculus I), Fall 2024

Each problem is worth 10 points, for a total of 50 points. You have 50 minutes to do the exam. Remember to *show your work* on all problems!

1. Compute the derivative $f'(x)$ for the following functions $f(x)$:

   (a) $f(x) = \frac{1}{\sqrt{x}}$

   (b) $f(x) = xe^x$

   (c) $f(x) = \cos(x^2 + 3x + 1)$

   (d) $f(x) = (\ln(5x))^2$

2. Consider the curve in the x, y-plane defined by the implicit equation $x^3 + xy + y^3 = 3$. Compute the slope of the tangent to this curve at the point $(x, y) = (1, 1)$.

3. Let $f: \mathbb{R} \to \mathbb{R}$ be $f(x) = xe^x$. Find all the critical points of f (i.e., points c with $f'(c) = 0$). For each critical point: is it the location of a local minimum, a local maximum, or neither?

4. As in the previous problem, let $f(x) = xe^x$. Compute the second derivative $f''(x)$. Then find the intervals where the graph of $f(x)$ is concave up, and those where it is concave down.

5. Use L'Hôpital's rule to compute the following limits:

   (a) $\displaystyle\lim_{x \to 0} \frac{\sin(x)}{-1 + e^{2x}}$

   (b) $\displaystyle\lim_{x \to 0^+} \frac{\ln(x)}{x^{-1}}$