Midterm #2 Study Guide
Math 156 (Calculus I), Fall 2024

1. Derivatives of basic functions [§3.1, 3.3, 3.6]
   (a) power functions: $d/dx(x^n) = nx^{n-1}$
   (b) exponential and logarithmic functions: $d/dx(e^x) = e^x$ and $d/dx(\ln(x)) = 1/x$
   (c) trigonometric functions: $d/dx(\sin(x)) = \cos(x)$ and $d/dx(\cos(x)) = -\sin(x)$
2. Rules for derivatives of combinations of functions [§3.1, 3.2, 3.4]
   (a) derivative is linear: $d/dx(\ a \cdot f(x) + b \cdot g(x)\ ) = a \cdot f'(x) + b \cdot g'(x)$ for $a, b \in \mathbb{R}$
   (b) product rule: $d/dx(\ f(x) \cdot g(x)\ ) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$
   (c) chain rule: $d/dx(\ f(g(x))\ ) = f'(g(x)) \cdot g'(x)$
   (d) quotient rule: $d/dx(\ f(x)/g(x)\ ) = \dfrac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2}$
   [*don't have to memorize quotient rule, it follows from other rules by writing* $f/g = f \cdot g^{-1}$]
3. Implicit differentiation and related rates [§3.5, 3.9]
   (a) for y defined implicitly via equation $p(x, y) = 0$, find dy/dx by taking d/dx of both sides, and use this to find the slope of the tangent at any point on the curve
   (b) if two quantities $f(t), g(t)$ are related, then their rates of change df/dt, dg/dt are related: like with implicit differentiation, just differentiate the relation between $f(t)$ and $g(t)$
4. Extreme values [§4.1, 4.3]
   (a) local versus absolute (global) minimum and maximum values, Extreme Value Theorem
   (b) the Closed Interval Method: extreme values of continuous f on closed interval must occur at endpoints or at critical points (values x where $f'(x) = 0$ or is not defined)
   (c) 1st and 2nd Derivative Tests for deciding if critical points are min.'s or max.'s
5. What derivatives tell us about shape of graph [§4.2, 4.3, 4.5]
   (a) $f'(x) > 0$ means f is increasing, $f'(x) < 0$ means f is decreasing
   (b) $f''(x) > 0$ means f is concave up (smile), $f''(x) < 0$ means f is concave down (frown)
6. L'Hôpital's rule [§4.4]
   (a) for indeterminate form limits (meaning "$\pm\frac{\infty}{\infty}$" or "$\frac{0}{0}$"), $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \lim\limits_{x \to a} \dfrac{f'(x)}{g'(x)}$