Math 4707: Summary of Class

5/3

Reminders: • Final due this Wednesday, 5/5.

• Today is the last day to submit SRTs (course feedback). Please get those in if you want your voice to be heard.

Today is the last day of class, and rather than present any new material, I thought we could try to summarize the main topics we covered this semester.

We started the semester discussing basic enumerative combinatorics: we talked about how to count things like subsets and permutations, and applied this kind of thinking to 'real-world' examples like Poker hands. These counting problems are closely related to Q's in probability ('what's the probability of a certain hand?'), and we explained how analysis of Pascal's triangle can be used to prove fundamental results in probability

like the Law of Large Numbers, in the case of independent coin flips. We learned two basic tools of discrete math: the Pigeonhole Principle and the Principle of Inclusion-Exclusion. We also discussed some famous number sequences like the Fibonacci Numbers and the Catalan numbers.

Then we moved on to graph theory. We studied Eulerian and Hamiltonian circuits (remember the Bridges of Königsberg?). We studied trees and especially their enumeration: both labeled (Cayley's formula) and unlabeled. We mentioned connections between graph theory and linear algebra.

Then we discussed some discrete optimization problems. We discussed the Minimum Spanning Tree problem (solved by Kruskal's algorithm). We discussed the Traveling Salesman Problem (hard!). We studied (perfect) matchings and Hall's

Marriage Theorem, and the more general Max-Flow Min-Cut problem (solved by the Ford-Fulkerson algorithm). Finally, we discussed the problem of stable marriages (solved by the Gale-Shapley algorithm).

Then we discussed some connections between geometry and combinatorics. We learned Euler's formula for planar graphs and polytopes. We studied coloring for graphs, especially coloring planar graphs (a.k.a. the 4 Color Thm.). Then we ended the semester with some more advanced topics related to planar graphs/coloring, including the chromatic polynomial.

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Q: what topics did you all

- like the most?
- dislike the most?
- wish we spent more/less time on?

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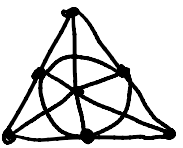
Some other fun topics in discrete math from the LVP textbook are:

- connections with number theory (Ch. 6)

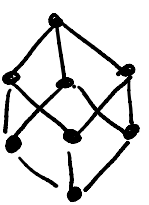
- projective planes and other combinatorial designs (Ch. 14).



- cryptography and other connections with theoretical computer science (Ch. 15).

A topic I love not in LVP is:

- the theory of partially ordered sets (posets) (see e.g. Stanton-White 'Constructive Combinatorics' or Stanley 'Algebraic Combinatorics')



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It's been a pleasure teaching all of you! I've been very impressed with your ability to keep up with all of this material! Continue to stay safe and have an excellent summer! ↑ get vaccinated! And good luck w/ all future endeavors!