

Finding a matching, Math 4707, Spring 2021

Your company has 10 employees $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}$ and 10 tasks to perform $\{y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}\}$, but each employee has a different set of tasks that they are capable of doing:

employee	tasks they can do
x_1	$\{y_2, y_4, y_5\}$
x_2	$\{y_1, y_2, y_3, y_5, y_6, y_7, y_8\}$
x_3	$\{y_2, y_5\}$
x_4	$\{y_2, y_4\}$
x_5	$\{y_4, y_5\}$
x_6	$\{y_1, y_3, y_4, y_6, y_7, y_8\}$
x_7	$\{y_2, y_7, y_8\}$
x_8	$\{y_4, y_7, y_8\}$
x_9	$\{y_5, y_7, y_8\}$
x_{10}	$\{y_7, y_8, y_9, y_{10}\}$

Match each employee to at most one task, so that different employees end up doing different tasks, in such a way that the maximum number of tasks are performed. Prove that your answer attains the maximum.

Bonus problem: come up with names for all the people and tasks (e.g., $x_1 =$ “Jarvis,” $y_1 =$ “swap the decaf and caf labels on the coffee pots,” $x_2 =$ “Mildred,” $y_2 =$ “make fax-machine noises in the printer room”).