

Principle of inclusion-exclusion worksheet, UMTYMP Advanced Topics, Fall 2020

1. How many anagrams of “COMMITTEE” have no double letters adjacent (i.e., no “MM”, “TT”, or “EE”)?
2. How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 20,$$

where each x_i is an integer with $0 \leq x_i \leq 8$? **Hint:** Let A_i be the integer solutions such that $x_i > 8$.

Let D_n denote the number of *derangements* in S_n , i.e., the number of permutations $p \in S_n$ with $p(i) \neq i$ for all $i \in [n]$. We used PIE to show

$$D_n = n! \cdot \sum_{k=0}^n (-1)^k \frac{1}{k!}$$

3. How many permutations $p \in S_n$ are there which in one-line notation never have the letter i immediately followed by $i + 1$, for $i = 1, 2, \dots, n - 1$?
4. Show that the answer you got to the last problem is the same as $D_n + D_{n-1}$. Can you give a bijective explanation for this?
5. Show that $D_{n+1} = n(D_n + D_{n-1})$ for $n \geq 1$ (where $D_0 = 1, D_1 = 0$).
6. Can you give a bijective proof for the last problem?