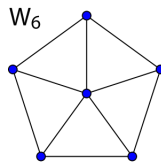


Planar graphs, UMTYMP Adv. Topics, Fall 2020

1. The *wheel graph* W_n on n vertices is obtained from the cycle graph C_{n-1} on $(n - 1)$ vertices by adding a new vertex adjacent to every other vertex; for instance W_6 looks like:

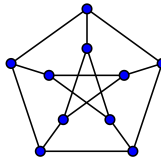


Show that W_n is always self-dual.

2. Show that the planar graphs corresponding to the icosahedron and dodecahedron are dual:



3. Explain why a planar graph G and its dual G^* have the same number of spanning trees.
4. Recall that the Petersen graph is



Find a subgraph of the Petersen graph that's a subdivision of $K_{3,3}$. Conclude that the Petersen graph is not planar. Can you find a subgraph of the Petersen graph that's a subdivision of K_5 ?

5. We showed that for a planar graph G , $\#E(G) \leq 3\#V(G) - 6$. We say G is *maximal planar* if this inequality is an equality. What do maximal planar graphs look like? Draw some.