

# **Dynamical algebraic combinatorics**

## ***[outline for notes as of 03/12/2022]***

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### **Chapter 0. Introduction**

- §0.1 Philosophy of dynamical algebraic combinatorics
- §0.2 Basic examples: rotation of subsets and multisets
- §0.3 Overview of rest of notes

### **Chapter 1. Tableaux**

- §1.1 Semistandard Young tableaux
- §1.2 Bender-Knuth involutions, promotion and evacuation
- §1.3 Reading words and Knuth equivalence
- §1.4 Jeu de taquin
- §1.5 Order of promotion for rectangular tableaux
- §1.6 Standard tableaux
- §1.7 Models for promotion for standard tableaux

### **Chapter 2. Posets**

- §2.1 Poset basics
- §2.2 Linear extensions, promotion and evacuation
- §2.3 Rectangles and triangles
- §2.4 Order ideals, rowmotion and toggles
- §2.5 Stanley–Thomas words
- §2.6 Piecewise-linear toggles and piecewise-linear rowmotion
- §2.7 Conjugacy of promotion and rowmotion
- §2.8 Rectangles and triangles again
- §2.9 Order polytopes

### **Chapter 3. Coxeter groups and root systems**

- §3.1 Coxeter group and root system basics
- §3.2 The symmetric group in detail
- §3.3 Root posets
- §3.4 Reduced words and the Edelman–Greene bijection
- §3.5 Absolute order and the Armstrong–Stump–Thomas bijection
- §3.6 Parabolic quotients
- §3.7 Minuscule posets
- §3.8 Toggling as reflection for minuscule posets

## **Chapter 4. Cyclic sieving**

- §4.1 The cyclic sieving phenomenon
- §4.2 Basic examples via linear algebra
- §4.3 General and special linear group representations
- §4.4 The Grassmannian and its coordinate ring
- §4.5 The “ $q=-1$ ” phenomenon via standard monomials
- §4.6 Cyclic sieving for SSYTs via canonical bases
- §4.7 Invariant tensors and Schur–Weyl duality
- §4.8 Cyclic sieving for SYTs via invariant tensors

## **Chapter 5. Invariance and homomesy**

- §5.1 Invariance and homomesy
- §5.2 Promotion invariants
- §5.3 Rowmotion invariants
- §5.4 Promotion homomesies
- §5.5 Rowmotion homomesies
- §5.6 Symmetry of Narayana numbers and rowvacuation

## **Appendix A. The RSK correspondence**

- §A.1 Row insertion description of RSK
- §A.2 Toggle description of RSK
- §A.3 Greene’s theorem
- §A.4 Symmetries of RSK
- §A.5 Sandwich theorem for rectangular tableaux

## **Appendix B. Birational dynamics**

- §B.1 Birational toggling and birational rowmotion
- §B.2 Nonintersecting lattice paths formula
- §B.3 Birational homomesies
- §B.4 Noncommutative dynamics

## **Appendix C. Beyond distributive lattices**

- §C.1 Rowmotion on distributive lattices
- §C.2 Semidistributive lattices
- §C.3 Trim lattices