MUST GIVE STEP BY STEP EXPLANATIONS TO GET CREDIT FOR ANSWERS.

No calculators or electronic devices are permitted.

## PART I: Do all three problems. EACH WORTH 24 POINTS.

- 1. Find the derivative of each functions by using the *limit definition* as indicated :
  - (a) Find f'(x) for  $f(x) = 4 2x + 7x^2$ .
  - (b) Find f'(2) for  $f(x) = \sqrt{x}$ .
- 2. Given the function  $f(x) = \frac{1}{3}x^3 x^2 3x + 2$ , with  $f'(x) = x^2 2x 3$  and f''(x) = 2x 2.
  - (a) Find the domain of f.
  - (b) Find the critical points.
  - (c) Find all intervals on which f is increasing, and all intervals on which f is decreasing.
  - (d) Find all relative extrema of f, or show that none exist.
  - (e) Find all intervals on which the graph of f is concave upward, and all intervals on which the graph of f is concave downward.
  - (f) Find all inflection points of the graph of f. If none exist, then note the fact.
  - (g) Sketch the graph of f, with appropriate labels, being sure to use the information from (a) to (f).
- 3. (a) The following chart lists a runner's speed at various times during a 60 second run. Estimate the distance traveled during this run, using right endpoints of each 10 second interval.

time $t$ in seconds	0	10	20	30	40	50	60
speed $v(t)$ in meters per second	8	12	14	10	12	10	8

- (b) i. Let the function f(x) be continuous on [0, 6]. Let  $\Delta x$  be the width of each subinterval in the partition  $P = x_0, x_1, ..., x_n$  of [0, 6], where  $0 = x_0 < x_1 < ... < x_n = 6$ . Let  $x_k^*$  be any sample point in the subinterval  $[x_{k-1}, x_k]$  for k = 1, ..., n. State the limit definition of the integral  $\int_0^6 f(x) dx$ .
  - ii. If f(x) = x<sup>2</sup> + x + 1, estimate \$\int\_{0}^{6}\$ f(x)dx\$ by using \$n = 3\$ subintervals, with the left endpoints of each interval as sample points. Draw a sketch of \$f(x)\$ and the approximating rectangles.
    iii. Is your estimate an over- or under-estimate of the true answer? Justify your answer.

## PART II: Choose any 8 problems. EACH WORTH 16 POINTS.

1. Given the piecewise function:

$$f(x) = \begin{cases} x^2 - 3x + 2, & \text{if } x < 1\\ 2x + 1, & \text{if } x \ge 1 \end{cases}$$

- (a) Continuity at x = 1: Using the definition of continuity, determine if the function f(x) is continuous at x = 1. Show your work and explain the steps involved.
- (b) Continuity on the interval  $(-\infty, \infty)$ : Determine if the function f(x) is continuous on the entire real line. Explain why or why not.
- 2. A cart on a flat, straight track begins moving forward at t = 0 seconds. For any time value  $0 \le t \le 5$ , the displacement of the cart relative to its initial position is  $s(t) = t^3 9t^2 + 24t$  meters.
  - (a) Find the instantaneous velocity of the cart at time t seconds.
  - (b) Determine the moments at which the velocity of the cart is zero meters per second.
  - (c) On which intervals is the cart moving forward and on which intervals is it moving backwards?
- 3. Use the linear approximation around  $x_0 = 16$  to find a rational number close to  $\sqrt{15}$ .

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4. A fixed quantity of a gas is being stored at a constant temperature so that the volume of gas in  $\text{cm}^3$ , V, and pressure it exerts on its container in kPa, P, are related by Boyle's Law:

$$PV = 10.$$

The container is being depressurized at a rate of -10 kPA/min. Determine the rate at which the volume of the gas is increasing the moment the pressure in the container is 100 kPa.

5. The table below shows values for a function f that is differentiable for all real numbers.

x	0	2	4	6	8	10
f(x)	-1	1	3	8	13	24
f'(x)	0	1	2	3	4	1

- (a) Identify one interval [a, b] in the table for which the slope of the secant line through two points on the graph of y = f(x) is 4. Briefly explain why f must be continuous on [a, b].
- (b) Explain why the Mean Value Theorem applies to f over [a, b].
- (c) Find the value of c such that a < c < b, for which the slope of the tangent line to the graph of y = f(x) is parallel to the secant line in part (a). Show all work.
- 6. Determine the equation of the line tangent to the curve

$$y^2(6-x) = x^3$$

at the point (3,3).

7. Evaluate the following limits using an appropriate method:

(a) 
$$\lim_{x \to 0} \left[ \frac{e^{6x} - 1}{\sin 2x} \right]$$
  
(b) 
$$\lim_{x \to \infty} \left[ \frac{3x^3 - 5x^2 + 2}{x^3 + x^2 - 4x} \right]$$
 and find the horizontal asymptote.

- 8. Use appropriate derivative rule/method to find the first derivative of each of the following functions and justify your solution.
  - a)  $f(x) = \cos(\tan(e^x + e^2))$ b)  $g(x) = \frac{\ln x + x^3}{\sec x}$ c)  $h(x) = \left(\sqrt[5]{x^3}\right)(\sin x)$
- 9. A rectangular prism has square bases, and the perimeter around each of the remaining sides is 16 inches. Find the dimensions of such a rectangular prism with a maximum volume.
- 10. Compute the following definite integrals by using the Fundamental Theorem of Calculus:

(a) 
$$\int_{-1}^{2} x^{3} + x - 2 \, dx$$
, (b)  $\int_{1}^{e^{2}} \frac{2}{x} \, dx$ , (c)  $\int_{0}^{\pi} \frac{1}{2} \sin(x) \, dx$ 

11. Evaluate the following indefinite integrals:

(a) 
$$\int 2x^3 - 3x^2 + 3x - 2 \, dx$$
, (b)  $\int 2e^x - \sin(x) \, dx$ , (c)  $\int \sqrt[3]{x} + \frac{2}{x^3} \, dx$ 

12. Use the *u*-substitution technique to evaluate the following indefinite integrals:

(a) 
$$\int \sqrt{5x-4} \, dx$$
, (b)  $\int \frac{\ln(x)}{x} \, dx$