MUST GIVE STEP BY STEP EXPLANATIONS TO GET CREDIT FOR ANSWERS.
No calculators or electronic devices are permitted.

PART I: *Do all three problems. EACH WORTH 24 POINTS.*

1. Do the following for the function $f(x) = (x+2)(x-2)^3$.

    (a) Show that $f'(x) = 4(x+1)(x-2)^2$ and $f''(x) = 12x(x-2)$.

    (b) Find the open interval(s) where f is increasing and/or decreasing.

    (c) Find any local maximum and/or minimum values of f.

    (d) Find the open interval(s) where f is concave up and/or concave down.

    (e) Find the coordinate(s) of any inflection points of f.

    (f) Use the information in parts (b) through (e) to sketch the graph of $y = f(x)$ that shows the information obtained in (b) through (e).

2. Consider $f(x) = \frac{1}{x-1}$.

    (a) Using the limit definition of the derivative, find the slope of the line tangent to $f(x)$ at $\left(3, \frac{1}{2}\right)$.

    (b) Give an equation for the tangent line at $\left(3, \frac{1}{2}\right)$.

3. The region R is bounded by the x-axis, the curve $y = x^2$, the line $x = 0$, and the line $x = 3$.

    (a) Approximate the area of R with the sum of the areas of the following three rectangles. These three rectangles are sitting on the x-axis, they have equal width, and their upper-right corners are on the curve $y = x^2$. Simplify your answer.

    (b) Approximate the area of R with the sum of the areas of the following n rectangles. These n rectangles are sitting on the x-axis, they have equal width, and their upper-right corners are on the curve $y = x^2$ $\left(\text{Hint: } \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}\right)$.

    (c) Evaluate $\int_0^3 x^2 dx$ by taking the limit of your previous answer as $n \to \infty$. Simplify your answer.

PART II: *Choose any 8 problems. EACH WORTH 16 POINTS.*

1. Let f be a function defined as follows:

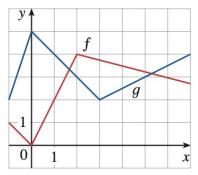$$f(x) = \begin{cases} x^2 + 3x, & \text{if } x < 1 \\ 4, & \text{if } x = 1 \\ 5x - 2, & \text{if } x > 1 \end{cases}$$

    (a) Find $\lim_{x \to 1^-} f(x)$. (b) Find $\lim_{x \to 1^+} f(x)$.

    (c) Find $\lim_{x \to 1} f(x)$ if it exists. If it does not exist, explain the reason.

    (d) Is f continuous at $x = 1$? Explain the reason to your answer.

2. The volume of a growing spherical cell is $V = \frac{4}{3}\pi r^3$. Find the instantaneous rate of change of the volume with respect to the radius when $r = 5\mu m$.

3. Find the horizontal and vertical asymptotes of the curve $f(x) = \frac{3x^2 + 4x + 8}{x^2 - 2x - 15}$. Justify your work for each by computing a limit.

4. (a) Find the linearization (linear approximation) of $f(x) = \frac{1}{2x-1}$ at $a = 0$.

    (b) Using your answer from part (a) approximate $f(0.1)$.

5. The base of a triangle is shrinking at a rate of 1 cm/min and the height of the triangle is increasing at a rate of 5 cm/min. Find the rate at which the area of the triangle changes when the height is 22 cm and the base is 10 cm.

6. The position $y = f(t)$ of a biker traveling along a straight road is a differentiable function of time t for $t > 0$. The table below shows the position of the biker at various times.

t (h)	2	4	6	8	10	12
y (mi)	2	32	22	25	30	34

(a) Identify one time interval $[a, b]$ in the table for which the average velocity is -5 miles per hour. Briefly explain why f must be continuous on $[a, b]$.

(b) Explain why the Mean Value Theorem applies to f over $[a, b]$.

(c) Show that there is some time c such that $a < c < b$, for which the instantaneous velocity $v(c) = -5$ miles per hour.

7. Consider the graphs of $f(x)$ and $g(x)$. Define $u(x) = f(x) \cdot g(x)$ and $v(x) = g(f(x))$. Find each of the following derivative values, if they exist. If a particular value does not exist, simply write DNE.

a) $u'(1)$
b) $u'(2)$

c) $v'(1)$
d) $v'(3)$

8. Evaluate the following limits using an appropriate method:

a) $\lim\limits_{x \to 5^+} \dfrac{8}{x - 5}$

b) $\lim\limits_{x \to 5^-} \dfrac{8}{x - 5}$

c) $\lim\limits_{x \to 1} \dfrac{2x^2 + 4x - 5}{5x - 3}$

d) $\lim\limits_{x \to 0} \dfrac{\tan 4x}{\sin 3x}$

9. For each of the following, find $\dfrac{dy}{dx}$.

a) $2x^2 + xy^2 + 2y^2 = 5$

b) $y = \dfrac{\sin(x^2)}{\arctan(x)} + e^{2x}$

c) $y = \sqrt{\dfrac{x - 1}{x^4 + 1}}$

10. Find the point on the curve $y = \dfrac{x^2}{6}$ which is closest to $(12, 3)$ using optimization method and justify your solution using the Second Derivative Test.

11. (a) Find an antiderivative of $f(x) = \dfrac{1}{\cos^2 x} + \dfrac{x^2 e^x + 4x^3}{x^2} - \dfrac{6}{x^2 + 1}$ with $F(0) = 2$.

(b) Find the following indefinite integrals:

i $\displaystyle\int e^x - \sin(x)\, dx$

ii $\displaystyle\int \sqrt{x} + \dfrac{1}{x^2}\, dx$

12. (a) Compute the following definite integrals by using the Fundamental Theorem of Calculus:

i. $\displaystyle\int_{-1}^{2} (x^3 + 2x - 1)\, dx$

ii $\displaystyle\int_{1}^{e} \dfrac{3}{x}\, dx$

iii $\displaystyle\int_{-\pi/2}^{\pi/2} \sin(x)\, dx$

(b) Use the u-substitution technique to find the following indefinite integrals:

i $\displaystyle\int \sqrt{3x + 2}\, dx$

ii $\displaystyle\int \dfrac{\ln(x)}{x}\, dx$