Final Exam Study Guide
Math 156 (Calculus I), Fall 2022

1. Basics (domain/range, what graph looks like, etc.) for standard functions [§1.1, 1.2, 1.4, 1.5]
   (a) algebraic functions: power functions (like x^3), root functions (like $\sqrt{x}$), polynomials (like $x^2 - 3x + 1$), rational functions (like $(x^2 - 1)/(x + 5)$)
   (b) trigonometric functions (like $\sin(x)$ and $\cos(x)$)
   (c) exponential functions (like e^x) and logarithmic functions (like $\ln(x)$)
   (d) piecewise functions (like absolute value $|x|$)
2. Algebraic operations on functions as geometric operations on graphs [§1.3]
   (a) translation (up/down & left/right), stretching (horiz. & vert.), reflection (over axes)
   (b) symmetry under these operations, especially even and odd functions
3. How to make new functions from old functions $f(x), g(x)$ [§1.3]
   (a) sum $(f + g)$, difference $(f - g)$, scaling (cf), product (fg), quotient (f/g)
   (b) composition of functions: $(f \circ g)(x) = f(g(x))$
4. Inverse functions $f = g^{-1}$ [§1.5]
   (a) especially exponential and logarithmic functions
   (b) graph of inverse function is reflection across line $y = x$
5. Intuitive definition of limit and basic reasons why a limit might not exist [§2.2]
   (a) $\lim_{x \to a} f(x) = L$ means can make $f(x)$ arbitrarily close to L by making $x \neq a$ close to a
   (b) one-sided limits $\lim_{x \to a^\pm} f(x)$: they must agree for usual (two-sided) limit to exist
6. How to compute limits using the limit laws [§2.3, 2.5]
   (a) sum $(f + g)$, difference $(f - g)$, scaling (cf), product (fg), quotient (f/g) limit laws
   (b) how to deal with "0/0" by cancelling factors
   (c) continuous functions (pushing limit thru, and direct substitution a.k.a. "plugging in")
7. Limits at infinity and limits equal to infinity [§2.2, 2.6]
   (a) limits at $\pm\infty$ = horizontal asymptotes
   (b) $\pm\infty$-valued limits = vertical asymptotes
8. The definition(s) of derivative [§2.1, 2.7, 2.8]
   (a) derivative as slope of the tangent to a curve at a point
   (b) derivative as a limit $f'(a) = \lim_{x \to a} (f(x) - f(a))/(x - a)$

9. Derivatives of basic functions [§3.1, 3.3, 3.6]
   (a) power functions: $d/dx(x^n) = nx^{n-1}$
   (b) exponential and logarithmic functions: $d/dx(e^x) = e^x$ and $d/dx(\ln(x)) = 1/x$
   (c) trigonometric functions: $d/dx(\sin(x)) = \cos(x)$ and $d/dx(\cos(x)) = -\sin(x)$
10. Rules for derivatives of combinations of functions [§3.1, 3.2, 3.4]
   (a) derivative is linear: $d/dx(\ a \cdot f(x) + b \cdot g(x)\ ) = a \cdot f'(x) + b \cdot g'(x)$ for $a, b \in \mathbb{R}$
   (b) product rule: $d/dx(\ f(x) \cdot g(x)\ ) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$
   (c) chain rule: $d/dx(\ f(g(x))\ ) = f'(g(x)) \cdot g'(x)$
   (d) quotient rule: $d/dx(\ f(x)/g(x)\ ) = (\ g(x) \cdot f'(x) - f(x) \cdot g'(x)\ )/(g(x))^2$
   [*don't have to separately memorize quotient rule, it follows from other rules*]
11. Implicit differentiation and related rates [§3.5, 3.9]
   (a) for y defined implicitly via equation $p(x, y) = 0$, find dy/dx by taking d/dx of both sides, and use this to find the slope of the tangent at any point on the curve
   (b) if two quantities $f(t), g(t)$ are related, then their rates of change df/dt, dg/dt are related: like with implicit differentiation, just differentiate the relation between $f(t)$ and $g(t)$
12. Linear approximation [§3.10]
   (a) tangent is best linear approximation to $f(x)$ near a point a: $f(x) \approx f(a) + (x - a) \cdot f'(a)$
13. Extreme values [§4.1, 4.3]
   (a) local versus absolute (global) minimum and maximum values, Extreme Value Theorem
   (b) the Closed Interval Method: extreme values of continuous f on closed interval must occur at endpoints or at critical points (values x where $f'(x) = 0$ or is not defined)
   (c) 1st and 2nd Derivative Tests for deciding if critical points are min.'s or max.'s
14. What derivatives tell us about shape of graph [§4.2, 4.3, 4.5]
   (a) $f'(x) > 0$ means f is increasing, $f'(x) < 0$ means f is decreasing
   (b) $f''(x) > 0$ means f is concave up (smile), $f''(x) < 0$ means f is concave down (frown)
15. L'Hôpital's rule [§4.4]
   (a) for indeterminate form limits (meaning "$\pm\frac{\infty}{\infty}$" or "$\frac{0}{0}$"), $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$
16. Anti-derivatives, a.k.a. indefinite integrals [§4.9, 5.4, 5.5]
   (a) basic anti-derivatives/indefinite integrals: $\int x^n\ dx = \frac{1}{n+1}x^{n+1} + C$, $\int e^x\ dx = e^x + C$, $\int \frac{1}{x}\ dx = \ln(x) + C$, $\int \sin(x)\ dx = -\cos(x) + C$, $\int \cos(x)\ dx = \sin(x) + C$
   (b) integral is linear: $\int a \cdot f(x) + b \cdot g(x)\ dx = a \int f(x)\ dx + b \int g(x)\ dx$ for $a, b \in \mathbb{R}$
   (c) the u-substitution technique: can treat the "dx" in an integral as a differential
17. Definite integrals [§5.1, 5.2, 5.3]
   (a) definite integral $\int_a^b f(x)\ dx$ as area under the curve $y = f(x)$ from $x = a$ to $x = b$, or more precisely as limit of "Riemann" (rectangle) sums $\lim_{n\to\infty} \sum_{i=1}^n f(x_i^*)\Delta x$
   (b) Fundamental Theorem of Calculus: $\int_a^b f(x)\ dx = F(b) - F(a) = \int f(x)\ dx\Big]_a^b$