Midterm #2 Study Guide
Math 157 (Calculus II), Spring 2024

1. More geometric applications of integrals [§8.1, 8.2]
   (a) Arc lengths of curves [§8.1]: length of $y = f(x)$ from $x = a$ to $x = b$ is $\int_a^b \sqrt{1 + (f'(x))^2}\, dx$.
   (b) Area of surface of revolution [§8.2]:
      i. for $y = f(x)$ from $x = a$ to $x = b$ rotated about x-axis, area is $\int_a^b 2\pi f(x) \sqrt{1 + (f'(x))^2}\, dx$;
      ii. for $y = f(x)$ from $x = a$ to $x = b$ rotated about y-axis, area is $\int_a^b 2\pi x \sqrt{1 + (f'(x))^2}\, dx$.
2. Parametrized curves [§10.1, 10.2]
   (a) Curve of form $x = f(t)$ and $y = g(t)$ for some auxiliary variable t ("time") [§10.1]
   (b) Slope of tangent [§10.2] to curve given by chain rule: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{g'(t)}{f'(t)}$
   (c) Arc length [§10.2] is $\int_a^b \sqrt{(\frac{dy}{dt})^2 + (\frac{dx}{dt})^2}\, dt = \int_a^b \sqrt{g'(t)^2 + f'(t)^2}\, dt$
3. Polar coordinates and polar curves [§10.3, 10.4]
   (a) Cartesian vs. polar [§10.3]: $(x, y) = (r\cos\theta, r\sin\theta)$ and $(r, \theta) = (\sqrt{x^2 + y^2}, \arctan(\frac{y}{x}))$
   (b) Area inside [§10.4] polar curve $r = f(\theta)$ for $\alpha \leq \theta \leq \beta$ is $\int_\alpha^\beta \frac{1}{2} r^2\, d\theta = \int_\alpha^\beta \frac{1}{2} f(\theta)^2\, d\theta$
   (c) Slope of tangent [§10.4] to polar curve $r = f(\theta)$ given by chain and product rules:
   $$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)} = \frac{f(\theta)\,\cos\theta + f'(\theta)\,\sin\theta}{f'(\theta)\cos\theta - f(\theta)\,\sin\theta}$$
   (d) Arc length [§10.4] of polar curve $r = f(\theta)$ is $\int_\alpha^\beta \sqrt{r^2 + (\frac{dr}{d\theta})^2}\, d\theta = \int_\alpha^\beta \sqrt{f(\theta)^2 + f'(\theta)^2}\, d\theta$