Midterm #2 Study Guide Math 157 (Calculus II), Spring 2025

- 1. More geometric applications of integrals [§8.1, 8.2]
 - (a) Arc lengths of curves [§8.1]: length of y = f(x) from x = a to x = b is $\int_a^b \sqrt{1 + f'(x)^2} dx$.
 - (b) Areas of surfaces of revolution [§8.2]:

i. for y = f(x) from x = a to x = b rotated about x-axis, area is $\int_a^b 2\pi f(x) \sqrt{1 + f'(x)^2} \, dx$.

- 2. Parametrized curves [§10.1, 10.2]
 - (a) Curve of form x = f(t) and y = g(t) for some auxiliary variable t ("time") [§10.1]
 - (b) Slope of tangent [§10.2] to curve given by chain rule: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{g'(t)}{f'(t)}$
 - (c) Arc length [§10.2] is $\int_a^b \sqrt{(\frac{dy}{dt})^2 + (\frac{dx}{dt})^2} dt = \int_a^b \sqrt{g'(t)^2 + f'(t)^2} dt$
- 3. Polar coordinates and polar curves [§10.3, 10.4]
 - (a) Cartesian vs. polar [§10.3]: $(x, y) = (r \cos \theta, r \sin \theta)$ and $(r, \theta) = (\sqrt{x^2 + y^2}, \arctan(\frac{y}{x}))$
 - (b) Area inside [§10.4] polar curve $r = f(\theta)$ for $\alpha \le \theta \le \beta$ is $\int_{\alpha}^{\beta} \frac{1}{2}r^2 d\theta = \int_{\alpha}^{\beta} \frac{1}{2}f(\theta)^2 d\theta$
 - (c) Slope of tangent [§10.4] to polar curve $r = f(\theta)$ given by chain and product rules:

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$