Midterm #3, 4/23
Math 157 (Calculus II), Spring 2025

Each problem is worth 10 points, for a total of 50 points. You have 50 minutes to do the exam. Remember to *show your work* on all problems!

1. For each of the following sequence limits, state the value of the limit or state that it diverges. Explain your answer.

   (a) $\lim_{n\to\infty} \dfrac{6n^2 - 2n - 1}{2n^2 + 2n + 1}$

   (b) $\lim_{n\to\infty} \sin(\dfrac{\pi}{n})$

   (c) $\lim_{n\to\infty} (-1)^n \cdot \dfrac{n}{n+1}$

   (d) $\lim_{n\to\infty} \ln(n)$

2. For each of the following series, decide (with explanation) whether it converges or diverges.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{4n}{2n-1}$ (**Hint**: check the limit of the terms.)

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{4}{2n-1}$ (**Hint:** compare to a series you know.)

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n+2}$ (**Hint:** it is an alternating series.)

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n + 3}{4^n + 4}$ (**Hint:** look at the ratio of successive terms.)

3. Consider the series $s = \displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n+1)^2}$. Let $s_n = \frac{1}{3^2} + \frac{1}{5^2} + \cdots + \frac{1}{(2n+1)^2}$ be its nth partial sum.

   (a) Compute the second partial sum s_2 as an estimate for the true value s of the series. (Do not worry about simplifying your answer.)

   (b) Let $R_2 = s - s_2$ be the corresponding remainder, i.e., the error of your estimate from part (a). Give an upper bound for R_2. (**Hint**: use an improper integral as the bound.)

4. Consider the function $f(x) = e^{-3x}$.

   (a) Express this function as a power series centered at zero: $f(x) = \sum_{n=0}^{\infty} c_n x^n$.

   (b) Determine the radius of convergence R of the power series you found in part (a).

5. Let $f\colon \mathbb{R} \to \mathbb{R}$ be a twice-differentiable function satisfying $f(0) = 4$, $f'(0) = 3$, and $f''(0) = 2$.

   (a) Write the degree two Taylor polynomial $T_2(x)$, centered at $x = 0$, for $f(x)$.

   (b) Use your answer in part (a) to estimate the value of $f(1)$. (You do not need to give any bounds on the error of your estimate.)