

Quiz #10, 4/11  
Math 157 (Calculus II), Spring 2024

Problem 1 is worth 6 points, and Problem 2 is worth 4 points, for a total of 10 points. Remember to *show your work* on all problems!

1. Consider the series  $s = \sum_{n=1}^{\infty} \frac{1}{n^3}$ . Let  $s_n = \sum_{k=1}^n \frac{1}{k^3}$  be the  $n$ th partial sum for this series.

- (a) Compute  $s_2$ , the second partial sum, as an estimate for the true value  $s$  of the series.
- (b) Let  $R_2 = s - s_2$  denote the error of your estimate. Compute upper and lower bounds on this error. **Hint:** recall that  $\int_{n+1}^{\infty} f(x) dx \leq R_n \leq \int_n^{\infty} f(x) dx$  for the appropriate  $f(x)$ .

2. For each of the following series, decide if it converges or diverges. Explain your answer.

(a)  $\sum_{n=1}^{\infty} \frac{4n^2 - n + 4}{3n^2 + 3n - 1}$  **(Hint:** look at the limit of the terms.)

(b)  $\sum_{n=1}^{\infty} \frac{1}{3^n + 1}$  **(Hint:** compare to a series you know.)

(c)  $\sum_{n=1}^{\infty} \frac{2}{2n - 1}$  **(Hint:** compare to a series you know.)

(d)  $\sum_{n=1}^{\infty} \frac{2}{2n^2 - 1}$  **(Hint:** compare to a series you know.)