Homework #11, Due: 4/17
Math 181 (Discrete Structures), Spring 2024

Problem 1 is worth 4 points, and Problem 2 is worth 6 points, for a total of 10 points. Remember to *show your work* and *explain your answers* on all problems!

1. In a standard deck of playing cards, cards have two qualities:

   - a *rank*: 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, or Ace;
   - a *suit*: Spades (♠), Hearts (♡), Diamonds (♢), or Clubs (♣).

   There are 13 ranks and 4 suits, and each combination of rank and suit appears exactly once. So there are a total of $13 \times 4 = 52$ cards. A *poker hand* consists of any 5 of these 52 cards. We saw in class that there are $C(52, 5) = 52!/(5! \cdot 47!) = 2,598,960$ different poker hands.

   (a) A poker hand is called *four of a kind* if it consists of all four cards of one rank, plus any other card. For instance: 8♠ 8♡ 8♢ 8♣ 3♢. How many four of a kind hands are there?

   (b) A poker hand is called a *full house* if it consists of three of the cards of one rank, and two of the cards of another rank. For instance: 5♠ 5♡ 5♣ J♡ J♢. How many full house hands are there?

2. (a) How many rearrangements of the word LOLLYPOP are there?

   (b) How many rearrangements of LOLLYPOP start with a Y or end with a P (or both)?
   Hint: remember the Principle of Inclusion and Exclusion!

   (c) How many rearrangements of LOLLYPOP have the two O's adjacent?
   Hint: treat the two O's as a single character "OO" to force them to be adjacent.