Homework #12, Due: 4/24
Math 181 (Discrete Structures), Spring 2024

Problem 1 is worth 6 points, Problem 2 is worth 2 points, and Problem 3 is worth 2 points, for a total of 10 points. Remember to *show your work* and *explain your answers* on all problems!

1. Recall that in class we proved the Binomial Theorem: $(x+y)^n = \sum_{k=0}^{n} C(n,k)x^k y^{n-k}$. Use the binomial theorem to prove the following identities for the binomial coefficients $C(n,k)$:

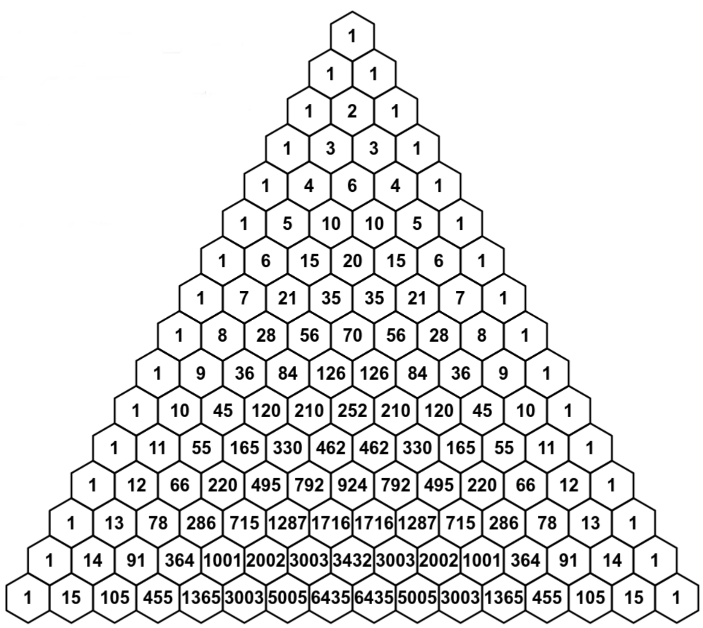
   (a) $\sum_{k=0}^{n} 2^k \cdot C(n,k) = 3^n$

   (b) $\sum_{k=0}^{n} (-1)^{n-k} \cdot 2^k \cdot C(n,k) = 1$

   (c) $\sum_{k=0}^{n} k \cdot C(n,k) = n \cdot 2^{n-1}$

   Hint: take the *derivative*, with respect to x, of the binomial theorem identity.

2. We saw how the $C(n,k)$ form Pascal's Triangle. Here are the first 16 rows of Pascal's Triangle:



   Fill in all of the odd values in the above triangle, and leave the even values unfilled. Describe the resulting pattern that you see.

3. Show that any subset of $X = \{1,2,3,4,5,6\}$ of size at least 4 contains a pair of elements whose sum is 7. **Hint**: use the Pigeonhole Principle, where the "holes" are the pairs of numbers in X summing to 7.