

Homework #5, Due: 2/14
Math 181 (Discrete Structures), Spring 2024

Problem 1 is worth 5 points, and Problem 2 is worth 5 points, for a total of 10 points. Remember to *show your work* and *explain your answers* on all problems!

1. Recall the set of integers is $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$. An integer x is called a *multiple* of an integer y if there is some integer z such that $x = z \times y$. For example, 15 is a multiple of 3 since $15 = 5 \times 3$.

Let $P(x, y)$ be the propositional formula “ x is a multiple of y ,” where the domain of discourse is the set $\mathbb{Z} \times \mathbb{Z}$ of pairs of integers. Write the meaning in English of the following propositions, and determine (with explanation) if they are true or false.

- (a) $\exists x \forall y P(x, y)$
 - (b) $\exists y \forall x P(x, y)$.
2. Give a direct proof of the following theorem about sets:
“For all sets X , Y , and Z , if $X \subseteq Y$ then $X \cup Z \subseteq Y \cup Z$.”