Homework #7, Due: 3/13
Math 181 (Discrete Structures), Spring 2024

Problem 1 is worth 5 points, and Problem 2 is worth 5 points, for a total of 10 points. Remember to *show your work* and *explain your answers* on all problems!

1. (a) Prove by induction that

$$\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$$

for all positive integers $n \geq 1$. (This kind of a sum is called a **geometric sum**; see Example 2.4.4 in the book.) Conclude that $\frac{1}{2^1} + \frac{1}{2^2} + \cdots + \frac{1}{2^n} < 1$ for all $n \geq 1$.

(b) Prove by induction that

$$\frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} + \cdots + \frac{n}{2^n} < 2$$

for all positive integers $n \geq 1$. **Hint**: notice the identity

$$\frac{1}{2^1} + \frac{2}{2^2} + \cdots + \frac{n+1}{2^{n+1}} = \left(\frac{1}{2^1} + \frac{1}{2^2} + \cdots + \frac{1}{2^{n+1}}\right) + \frac{1}{2}\left(\frac{1}{2^1} + \frac{2}{2^2} + \cdots + \frac{n}{2^n}\right)$$

and use the result in part (a).

2. By experimenting with small values, guess a formula for

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)}$$

and then use induction to prove that your formula is correct for all positive integers $n \geq 1$.