

Homework #8, Due: 3/20
Math 181 (Discrete Structures), Spring 2024

Problem 1 is worth 3 points, Problem 2 is worth 3 points, and Problem 3 is worth 4 points, for a total of 10 points. Remember to *show your work* and *explain your answers* on all problems!

1. Let $X = \{1, 2, 3\}$ and $Y = \{w, x, y, z\}$. For each of the following subsets of $X \times Y$, does it correspond to a function $f: X \rightarrow Y$? Explain.

(a) $\{(3, w), (1, w), (2, x)\}$

(b) $\{(1, w), (3, z)\}$

(c) $\{(1, y), (2, x), (3, z), (1, x)\}$

2. Let $X = \{0, 1, 2, 3, 4, 5\}$. Let $f: X \rightarrow X$ be defined by

$$f(x) = 2x \pmod{6},$$

for all $x \in X$. Draw the arrow diagram of the function f . Is f one-to-one? Is f onto?

3. Let $f: \mathbb{Z} \rightarrow \mathbb{Z}$ and $g: \mathbb{Z} \rightarrow \mathbb{Z}$ be defined by

$$f(n) = 2n - 1, \quad g(n) = 3n + 1,$$

for all $n \in \mathbb{Z}$. Write the formulas defining each of the following functions from \mathbb{Z} to \mathbb{Z} , which are various compositions of f and g .

(a) $f \circ f$

(b) $f \circ g$

(c) $g \circ f$

(d) $g \circ g$