Homework #9, Due: 3/27
Math 181 (Discrete Structures), Spring 2024

Problem 1 is worth 4 points, Problem 2 is worth 2 points, and Problem 3 is worth 4 points, for a total of 10 points. Remember to *show your work* and *explain your answers* on all problems!

1. Let $X = \{a, b\}$ and recall that X^* denotes the set of all strings over the alphabet X. Define a function $f \colon X^* \to X^*$ by letting $f(\alpha)$ be the result of simultaneously replacing each a with a b, and each b with an a, in the string $\alpha \in X^*$. For instance $f(aab) = bba$.

   (a) Write what $f(a)$, $f(bb)$, $f(baba)$, and $f(\lambda)$ are. (Recall $\lambda \in X^*$ denotes the null string.)

   (b) Recall that for strings $\alpha, \beta \in X^*$, we use $\alpha\beta$ to mean the concatenation of α and β. Express $f(\alpha\beta)$ in terms of $f(\alpha)$ and $f(\beta)$.

   (c) What is $f(f(\alpha))$ for a string $\alpha \in X^*$?

   (d) Is f one-to-one? Is f onto? Explain.

2. Let R be the relation on $\{1, 2, 3, 4\}$ given by $R = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$. Draw the digraph representation of R. Also draw the digraph representation of R^{-1} (the inverse relation).

3. Let R be the relation on the integers $\mathbb{Z}$ where $(x, y) \in R$ if and only if $x - y$ is even.

   (a) Is R reflexive? Explain.

   (b) Is R symmetric? Explain.

   (c) Is R anti-symmetric? Explain.

   (d) Is R transitive? Explain.