Midterm #1, 2/21
Math 181 (Discrete Structures), Spring 2024

Each problem is worth 10 points, for a total of 50 points. You have 50 minutes to do the exam. Remember to *show your work* and *explain your answers* on all problems!

1. In this problem, let the universal set be $U = \{1, 2, 3, 4, 5\}$ and let $A = \{1, 3, 5\}, B = \{2, 5\}$. Write the following sets:

   (a) $(A \cap B)^c$ (this is also written $\overline{A \cap B}$ in the book)

   (b) $B \setminus A$ (this is also written $B - A$ in the book)

   (c) $(A \setminus B) \cup (B \setminus A)$

2. Write the truth tables for the compound propositions $p \to q$ and $q \vee \neg p$. What does this tell you about the relationship between these two propositions?

3. Convert this argument to a symbolic form, and determine (with explanation) if it is valid:

   Hypotheses: If it's sunny out, then I walk to work. I do not walk to work.
   Conclusion: It's not sunny out.

4. Let $P(x)$ be the propositional formula "$x^2 \geq 1$," where the domain of discourse is the set $\mathbb{R}$ of all real numbers. Write the proposition "$\neg(\forall x\ P(x))$" in English words, and determine (with explanation) if it is true or false.

5. Give a direct proof of the following theorem about integers:
   "The product of an even integer and any integer is an even integer."