Midterm #2, 4/10
Math 181 (Discrete Structures), Spring 2024

Each problem is worth 10 points, for a total of 50 points. You have 50 minutes to do the exam. Remember to *show your work* and *explain your answers* on all problems!

1. Prove the following theorem:
   "If the product of two integers is even, then at least one of these two integers must be even."
   Use proof by contrapositive or proof by contradiction.

2. Prove by induction that $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ for all integers $n \geq 1$. (The left-hand side of the identity is the sum of all odd positive integers less than or equal to $2n - 1$.)

3. Let $X = \{0, 1, 2, 3\}$ and define a function $f \colon X \to X$ by $f(x) = (3x+2) \mod 4$ for all $x \in X$. Draw the arrow diagram of f. Is f one-to-one? Is f onto?

4. Let $X = \{a, b, c\}$, and consider the set $X^* \setminus \{\lambda\}$ of non-null strings over the alphabet X. Let R be the relation on this set of strings where for $\alpha, \beta \in X^* \setminus \{\lambda\}$ we have $\alpha \; R \; \beta$ if and only if α and β have the same first letter. For example, $abc \; R \; acabb$ and $bb \; R \; bca$ and $c \; R \; caa$. Explain why R is an equivalence relation, and describe all the equivalence classes of R.

5. Let $A = \{1, 2\}$ and $C = \{1, 2, 3, 4, 5, 6\}$. How many sets B with $A \subseteq B \subseteq C$ are there? Explain your answer, for instance by referencing a counting principle.