

Midterm #2 Study Guide

Math 181 (Discrete Structures), Spring 2024

1. Indirect proofs [§2.2]

- (a) proof by contrapositive: to prove $p \rightarrow q$, prove $\neg q \rightarrow \neg p$ instead
- (b) proof by contradiction: assume negation of statement, and deduce contradiction ($r \wedge \neg r$)

2. Mathematical induction [§2.4, 2.5]

- (a) basic structure of inductive proofs: base case $P(1)$, and induction step $P(n) \rightarrow P(n+1)$
- (b) proving $\forall(n \in \mathbb{Z}_{>0}) P(n)$ by induction, especially when $P(n)$ is an algebraic formula
- (c) finding patterns to guess formulas involving n which can then be proved by induction

3. Functions [§3.1]

- (a) ways to view a function $f: X \rightarrow Y$: rule to convert input $x \in X$ to output $y = f(x) \in Y$; set of ordered pairs (x, y) ; arrow diagram from X to Y
- (b) one-to-one, onto, and bijective functions
- (c) composition of functions, and inverse functions
- (d) modular arithmetic functions like $f(x) = x \pmod n$

4. Sequences and strings [§3.2]

- (a) finite and infinite sequences: ordered list of elements of some set
- (b) set of strings X^* on a finite alphabet X , the null string $\lambda \in X^*$, concatenation of strings
- (c) subsequences (not necessarily consecutive) versus substrings (consecutive)

5. Relations [§3.4, 3.5]

- (a) digraph representation of a relation R on a set X
- (b) properties that R can have: reflexive, symmetric, anti-symmetric, transitive
- (c) partial order (reflexive, anti-symmetric, transitive): way to “compare” things in X
- (d) equivalence relation (reflexive, symmetric, transitive): way to say certain things in X are “the same”; corresponds to a partition of X into equivalence classes

6. Basic counting principles [§6.1]

- (a) multiplication principle: total # of possibilities = product of # of choices at each step
- (b) addition principle: size of union of *disjoint* sets is sum of sizes of the sets
- (c) principle of inclusion and exclusion: $\#(X \cup Y) = \#X + \#Y - \#(X \cap Y)$