

Principle of inclusion-exclusion,
Math 4707, Spring 2021

1. How many anagrams of “BALLOON” have no double letters adjacent (i.e., no “LL” nor “OO”)?
2. How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 20,$$

where each x_i is an integer with $0 \leq x_i \leq 8$? **Hint:** Let A_i be the integer solutions such that $x_i > 8$.

Let D_n denote the number of *derangements* in S_n , i.e., the number of permutations $p \in S_n$ with $p(i) \neq i$ for all $i \in [n]$. We used PIE to show

$$D_n = n! \cdot \sum_{k=0}^n (-1)^k \frac{1}{k!}$$

3. How many permutations $p \in S_n$ are there which in one-line notation never have the letter i immediately followed by $i + 1$, for $i = 1, 2, \dots, n - 1$?
4. Show that the answer you got to the last problem is the same as $D_n + D_{n-1}$.
Challenge: can you give a *bijective* explanation for this?
5. Show that $D_{n+1} = n(D_n + D_{n-1})$ for $n \geq 1$ (where $D_0 = 1, D_1 = 0$).
Challenge: can you give a *bijective* explanation for this?