

Poker hand probabilities, Math 4707, Spring 2021

In a standard deck of cards, there are 4 possible suits (clubs, diamonds, hearts, spades), and 13 possible values (2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, Ace). Let A, J, Q, K represent Ace, Jack, Queen and King, respectively. Every card has a suit and value, and every combination is possible. Hence a standard deck contains $13 \cdot 4 = 52$ cards. A “poker hand” consists of 5 unordered cards from a standard deck of 52. There are $\binom{52}{5} = 2,598,960$ possible poker hands. Let us now calculate the probability of each of the standard kinds of poker hands.

Royal Flush. This hand consists of values 10, J, Q, K, A, all of the same suit. There are _____ royal flushes, for a probability of 0.000154%.

Straight Flush. A straight flush consists of five cards with values in a row, all of the same suit. Ace may be considered as high or low, but not both. (For example, A, 2, 3, 4, 5 is a straight, and 10, J, Q, K, A is a straight, but Q, K, A, 2, 3 is not a straight.) Excluding the royal flushes we already counted, there are _____ straight flushes, for a probability of 0.00139%.

Four of a Kind. This hand consists of four cards of one value, and a fifth card of a different value. There are _____ four of a kinds, for a probability of 0.0240%.

Full House. This hand consists of three cards of one value, and two cards of a different value. There are _____ full houses, for a probability of 0.144%.

Flush. A flush consists of five cards, all of the same suit, *with not all values in a row* (to exclude the straight flushes we already counted). There are _____ flushes, for a probability of 0.197%.

Straight. A straight consists of five values in a row, *not all of the same suit* (to exclude the straight flushes we already counted). There are _____ straights, for a probability of 0.392%.

Three of a Kind. This hand consists of three cards of one value, and two more cards, each of different values. There are _____ three of a kinds, for a probability of 2.11%.

Two Pairs. This hand consists of two pairs of different values, and a fifth card of another different value. There are _____ two of a kinds, for a probability of 4.75%.

One Pair. This hand consists of a pair of one value, and three additional cards, each of different values. There are _____ pairs, for a probability of 42.3%.

Nothing. This is any hand that does not fall into one of the above categories. There are _____ nothings, for a probability of 50.1%.