Walks and circuits, Math 4707, Spring 2021

An Eulerian walk in a graph G is a walk that crosses each edge exactly once. An Eulerian circuit is an Eulerian walk that's a circuit (a closed walk).

1. For each of the following graphs: does the graph have an Eulerian walk? Does it have an Eulerian circuit?



A Hamiltonian path in a graph G is a path that visits every vertex exactly once. A Hamiltonian cycle in a graph G is a cycle that visits every vertex exactly once, except that, being a cycle, it ends at the vertex it starts at.

2. For each of the following graphs: does the graph have a Hamiltonian path? Does it have a Hamiltonian cycle?



3. Let G be a connected graph. Fill in the blanks:

- If G has 0 vertices with odd degree, then it has an _____.
- If G has exactly 2 vertices with odd degree, then it has an ______ but not an ______.
- If G has more than 2 vertices with odd degree, then it doesn't even have an _____.

Explain why these 3 cases cover all the possibilities for the number of odd degree vertices in a graph G.